COURSE GUIDE

Functional Analysis (270113B)

Approval date: 22/06/2023

Grado (Bachelor's Degree)	Grado en Matemáticas	Branch	Sciences
Module	Análisis Matemático	Subject	Análisis Funcional

Year of study	3º	Semester	1º	ECTS Credits	6	Course type	Compulsory course

PREREQUISITES AND RECOMMENDATIONS

Students are recommended to have successfully completed the basic and obligatory subjects of the first two years of the Degree in Mathematics.

BRIEF DESCRIPTION OF COURSE CONTENT (According to the programme's verification report)

- Normed spaces
- Hilbert spaces
- Compact linear operators on Hilbert spaces
- Duality in normed spaces
- Weak topologies

SKILLS

GENERAL SKILLS

- CG01 - Poseer los conocimientos básicos y matemáticos de las distintas materias que, partiendo de la base de la educación secundaria general, y apoyándose en libros de texto avanzados, se desarrollan en esta propuesta de título de Grado en Matemáticas
- CG02 - Saber aplicar esos conocimientos básicos y matemáticos a su trabajo o vocación de una forma profesional y poseer las competencias que suelen demostrarse por medio de la elaboración y defensa de argumentos y la resolución de problemas dentro de las Matemáticas y de los ámbitos en que se aplican directamente
- CG03 - Saber reunir e interpretar datos relevantes (normalmente de carácter matemático) para emitir juicios que incluyan una reflexión sobre temas relevantes de índole social, científica o ética
- CG04 - Poder transmitir información, ideas, problemas y sus soluciones, de forma escrita u oral, a un público tanto especializado como no especializado
- CG06 - Utilizar herramientas de búsqueda de recursos bibliográficos

SUBJECT-SPECIFIC SKILLS

- CE01 - Comprender y utilizar el lenguaje matemático. Adquirir la capacidad de enunciar proposiciones en distintos campos de las matemáticas, para construir demostraciones y para transmitir los conocimientos matemáticos adquiridos
- CE02 - Conocer demostraciones rigurosas de teoremas clásicos en distintas áreas de Matemáticas
- CE03 - Asimilar la definición de un nuevo objeto matemático, en términos de otros ya conocidos, y ser capaz de utilizar este objeto en diferentes contextos
- CE04 - Saber abstraer las propiedades estructurales (de objetos matemáticos, de la realidad observada, y de otros ámbitos) y distinguirlas de aquellas puramente accidentales, y poder comprobarlas con demostraciones o refutarlas con contraejemplos, así como identificar errores en razonamientos incorrectos
- CE05 - Resolver problemas matemáticos, planificando su resolución en función de las herramientas disponibles y de las restricciones de tiempo y recursos
- CE06 - Proponer, analizar, validar e interpretar modelos de situaciones reales sencillas, utilizando las herramientas matemáticas más adecuadas a los fines que se persigan
- CE07 - Utilizar aplicaciones informáticas de análisis estadístico, cálculo numérico y simbólico, visualización gráfica, optimización u otras para experimentar en matemáticas y resolver problemas

TRANSFERABLE SKILLS

- CT01 - Desarrollar cierta habilidad inicial de "emprendimiento" que facilite a los titulados, en el futuro, el autoempleo mediante la creación de empresas
- CT02 - Fomentar y garantizar el respeto a los Derechos Humanos y a los principios de accesibilidad universal, igualdad ante la ley, no discriminación y a los valores democráticos y de la cultura de la paz

LEARNING OUTCOMES

General goals:

- Abstraction capacity for the study of standard problems in Mathematical Analysis from the point of view of functional analysis, by understanding the advantages of functional analysis methods for solving various problems.
- Becoming familiar with some spaces of functions that are commondly employed in Mathematical Analysis and in its applications, like spaces of continuous (respectively, differentiable, analytic or harmonic, integrable, etc.) functions.
- Preparation for further studies, both in Mathematical Analysis and in other branches of Mathematics.

Specific goals:

- Employ the concepts of convergent sequence and Cauchy sequence in normed spaces, and understanding the notion of completeness.
- Being able to use the Hölder and Minkowski inequalities in concrete cases.
- Getting the skills to prove the continuity of some linear operators between normed spaces and compute their norms.
- Describe the dual space of some normed spaces.
- Being able to handle the orthogonal projection of a Hilbert space onto a closed subspace, and to compute the Fourier expansion with respect to the trigonometric system and Bessel's (in)equality.
- Study some examples of compact operators related to differential and integral equations.

- Compute the adjoint operator of some concrete operators on Hilbert spaces.
- Applications of the Hahn-Banach theorem.
- Check the reflexivity of some Banach spaces.
- Apply the Uniform boundedness principle.
- Formulate and understand the weak and weak-* topologies on some Banach spaces.

PLANNED LEARNING ACTIVITIES

THEORY SYLLABUS

Chapter 1. Normed spaces.
Basic concepts and examples. Completeness. Banach's fixed point theorem. Continuous linear operators and functionals. Complemented subspaces. Quotients of normed spaces. Dual space of a normed space. Examples. Finite-dimensional normed spaces.
Chapter 2. Hilbert spaces.
Scalar Products and (pre-)Hilbert Spaces. Projection onto a closed convex set. Orthogonal projection theorem. Riesz-Fréchet representation theorem. Dual of a Hilbert space. Orthonormal bases. Operators on Hilbert spaces. Compact operators.
Chapter 3. Hahn-Banach theorem and Applications.
Analytical and geometric versions of the Hahn-Banach theorem. Separation of convex sets. Duality in normed spaces. Bidual of a normed space. Reflective spaces. The weak topology of a normed space and the weak-* topology of its dual space.
Chapter 4: The uniform boundeness principle and the closed graph theorem.
Baire category theorem. Banach-Steinhaus theorem. Applications. Open mapping theorem or Banach-Schauder theorem. Closed graph theorem. Applications.

PRACTICAL SYLLABUS

The practices of this course will consist of workshops to solve exercises and problems related to the theoretical contents.

RECOMMENDED READING

ESSENTIAL READING

BREZIS, H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations. Springer, 2011.
CONWAY, J.K.: A Course in Functional Analysis, Springer-Verlag. New York, 1990.
MacCLUER, B.D.: Elementary Functional Analysis. Springer, 2009.
KADETS, V.; A Course in Functional Analysis and Measure Theory, Springer, 2018.
RINNE, P.R.; YOUNGSON,M.A.: Linear Functional Analysis. 2nd ed. Springer, 2008.
WILLEM, M.: Functional Analysis. Fundamentals and Applications. Birkhäuser, 2010.

COMPLEMENTARY READING

BERBERIAN, S.K.: Lectures in Functional Analysis and Operator Theory.Springer-Verlag, New York, 1974.
DIEUDONNÉ,J.: History of Functional Analysis. North-Holland,Amsterdam,1981.

RUDIN, W.: Functional Analysis. McGraw-Hill, New York, 1973.
Lecture notes by Prof. Rafael Payá: https://www.ugr.es/~rpaya/cursosanteriores.htm (in Spanish)
Lecture notes by Prof. Javier Pérez: https://www.ugr.es/~fjperez/apuntes.html (in Spanish)

RECOMMENDED LEARNING RESOURCES/TOOLS

http://mathworld.wolfram.com/topics/FunctionalAnalysis.html
https://mathshistory.st-andrews.ac.uk/
https://encyclopediaofmath.org/wiki/Functional_analysis

TEACHING METHODS

- MD01 - Lección magistral/expositiva
- MD03 - Resolución de problemas y estudio de casos prácticos
- MD06 - Análisis de fuentes y documentos
- MD07 - Realización de trabajos en grupo
- MD08 - Realización de trabajos individuales

ASSESSMENT METHODS (Instruments, criteria and percentages)

ORDINARY EXAMINATION DIET

A continuous assessment method will be followed, which will consist of two partial tests and a final exam. Lectures attendance is voluntary. The partial tests, to be carried out on a date that will be set well in advance, will comprise theoretical and practical contains and will be carried out in person. Each of these two tests will contribute 25% of the final grade. For the overall assessment of the assimilated knowledge and the skills acquired by the students, a final exam will be held on the date officially established for it. This exam will comprise theoretical and practical contains from all the contains of the subject. The final test must be done in person. The score of this exam will represent 50% of the final grade. The final grade will be expressed numerically as a result of the indicated weighting.

EXTRAORDINARY EXAMINATION DIET

There will be a single final exam of a theoretical and practical nature, which will include all the contents of the subject. It must be done in person. The score obtained in this exam will represent 100% of the grade.

SINGLE FINAL ASSESSMENT (evaluación única final)

Students who, following the regulations of the UGR in the terms and deadlines required therein, take advantage of the "Single Final Assessment modality" for their evaluation, will take a single final exam that will consist of theory and problems covering all the contents of the subject. It must be done in person. The grade obtained in said exam will represent 100% of the final grade.