Approval date: **11/07/2022**

COURSE GUIDE

Numerical Analysis in Partial Differential Equations (27011D3)

Grado (Bachelor's Degree)	Grado en Matemáticas	**Branch**	Sciences
Module	Complementos de Matemática Aplicada	**Subject**	Análisis Numérico de Ecuaciones en Derivadas Parciales

Year of study	4º	**Semester**	2º	**ECTS Credits**	6	**Course type**	Elective course

PREREQUISITES AND RECOMMENDATIONS

It is recommended to have studied (or be studying) the subjects corresponding to the modules "Ordinary and Partial Differential Equations", "Numerical Methods" and "Modelling and optimisation"

BRIEF DESCRIPTION OF COURSE CONTENT (According to the programme's verification report)

Partial differential equations (PDEs) numerical aproximation: finite difference, finite element and variational methods.

SKILLS

GENERAL SKILLS

- CG01 - Poseer los conocimientos básicos y matemáticos de las distintas materias que, partiendo de la base de la educación secundaria general, y apoyándose en libros de texto avanzados, se desarrollan en esta propuesta de título de Grado en Matemáticas
- CG02 - Saber aplicar esos conocimientos básicos y matemáticos a su trabajo o vocación de una forma profesional y poseer las competencias que suelen demostrarse por medio de la elaboración y defensa de argumentos y la resolución de problemas dentro de las Matemáticas y de los ámbitos en que se aplican directamente
- CG03 - Saber reunir e interpretar datos relevantes (normalmente de carácter matemático) para emitir juicios que incluyan una reflexión sobre temas relevantes de índole social, científica o ética
- CG04 - Poder transmitir información, ideas, problemas y sus soluciones, de forma escrita u oral, a un público tanto especializado como no especializado
- CG05 - Haber desarrollado aquellas habilidades de aprendizaje necesarias para emprender estudios posteriores con un alto grado de autonomía

- CG06 - Utilizar herramientas de búsqueda de recursos bibliográficos

SUBJECT-SPECIFIC SKILLS

- CE01 - Comprender y utilizar el lenguaje matemático. Adquirir la capacidad de enunciar proposiciones en distintos campos de las matemáticas, para construir demostraciones y para transmitir los conocimientos matemáticos adquiridos
- CE02 - Conocer demostraciones rigurosas de teoremas clásicos en distintas áreas de Matemáticas
- CE03 - Asimilar la definición de un nuevo objeto matemático, en términos de otros ya conocidos, y ser capaz de utilizar este objeto en diferentes contextos
- CE04 - Saber abstraer las propiedades estructurales (de objetos matemáticos, de la realidad observada, y de otros ámbitos) y distinguirlas de aquellas puramente accidentales, y poder comprobarlas con demostraciones o refutarlas con contraejemplos, así como identificar errores en razonamientos incorrectos
- CE05 - Resolver problemas matemáticos, planificando su resolución en función de las herramientas disponibles y de las restricciones de tiempo y recursos
- CE06 - Proponer, analizar, validar e interpretar modelos de situaciones reales sencillas, utilizando las herramientas matemáticas más adecuadas a los fines que se persigan
- CE07 - Utilizar aplicaciones informáticas de análisis estadístico, cálculo numérico y simbólico, visualización gráfica, optimización u otras para experimentar en matemáticas y resolver problemas
- CE08 - Desarrollar programas que resuelvan problemas matemáticos utilizando para cada caso el entorno computacional adecuado

TRANSFERABLE SKILLS

- CT01 - Desarrollar cierta habilidad inicial de "emprendimiento" que facilite a los titulados, en el futuro, el autoempleo mediante la creación de empresas
- CT02 - Fomentar y garantizar el respeto a los Derechos Humanos y a los principios de accesibilidad universal, igualdad ante la ley, no discriminación y a los valores democráticos y de la cultura de la paz

LEARNING OUTCOMES

- Understanding, analysis and implementation of methods for PDE's approximate resolution.
- Critical assessment of the obtained results.
- Knowledge of the basic numerical methods considered.

PLANNED LEARNING ACTIVITIES

THEORY SYLLABUS

Topic 1. Introduction to finite difference methods

1.1. Brief reminder: finite difference approximations for derivatives. Matrix norms. Iterative methods for linear systems.
1.2. Boundary value problems: consistency, stability and convergence.

1.3. Elliptic boundary problems.
1.4. Evolution problems: diffusion and hyperbolic equations.
1.4.1. Finite difference schemes for the heat equation.
1.4.2. Schemes for the advection equation.
1.4.3. Difference schemes for the wave equation.

Topic 2. The finite element method.

2.1. Solving PDEs by variational techniques: weak formulation and Lax-Milgram Th.
2.2. Brief reminder: spline interpolation. Introduction: 1D finite element method.
2.3. Finite element spaces. Cea Lemmas. The interpolation operator.
2.4. Solving evolution problems.

PRACTICAL SYLLABUS

P1. EDP solution approximation by using finite difference schemes.

P2. EDP solution approximation by using finite element schemes.

Specific scientific computing languages, to be announced by the teachers, will be employed.

RECOMMENDED READING

ESSENTIAL READING

1. R. J. Leveque, Finite Difference Methods for Ordinary and Partial Differential Equations, SIAM, Philadelphia, 2007.
2. J. C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, SIAM Philadelphia, 2004.
3. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.
4. P. A. Raviart, J. M. Thomas, Introduction a l'Analyse Numerique des Equations aux Derivees Partielles, Masson, París, 1988.
5. C. Johnson, Numerical solutions of partial differential equations by the finite element method, Cambridge University Press, 1987.
6. A. Quarteroni, Numerical Models for Differential Problems, Series: MSA, Vol 2, 2009.

COMPLEMENTARY READING

1. The Octave Project Developers, GNU Octave (version 7.1.0), consultado en Mayo 2022.
2. A. Delgado, J.J. Nieto, A. M. Robles, O. Sánchez, Métodos Numéricos básicos con Octave, Ed. Técnica AVICAM (Fleming), Granada 2016.
3. F. Hecht, FreeFem, version 4.11, consultado en Mayo 2022.
4. G. D. Smith. Numerical solution of partial differential equations: finite difference methods, Clarendon Press, Oxford, 1985.
5. A. Ern, J. L. Guermond, Theory and Practice of Finite Elements, Springer-Verlag, New York, 2004.
6. S. Brenner, R. Scott, The mathematical theory of Finite Element Methods, Springer New

York, New York, 2008
7. B. Lucquin, O. Pironneau, Introduction au calcul scientifique, Masson, París, 1996.
8. A. Quarteroni, R. Sacco, F. Saleri, Numerical Mathematics, Text in Applied Mathematics, V. 37, Springer-Verlag, New-York, 2007.
9. W. Gautschi, Numerical Analysis, Birkhäuser-Boston, 2012.

RECOMMENDED LEARNING RESOURCES/TOOLS

The information about this course, course materials and student's marks will be hosted on the platform PRADO: https://prado.ugr.es.

- Universidad de Granada webpage: http://www.ugr.es
- Universidad de Granada library webpage: https://biblioteca.ugr.es/
- Departamento de Matemática Aplicada webpage: https://mateapli.ugr.es/

Webpages of experts on the field:

- Randall Leveque: Randy LeVeque (washington.edu)
- Alfio Quarteroni: http://cmcs.epfl.ch/people/quarteroni CMCS | EPFL

Official webpages of software employed in the course:

- Octave: GNU Octave
- Python: Welcome to Python.org and extra scientific computing packages (SciPy, NumPy, SymPy, Matplotlib, etc): SciPy.org
- Freefem: A high level multiphysics finite element software. https://freefem.org/

TEACHING METHODS

- MD01 - Lección magistral/expositiva
- MD02 - Sesiones de discusión y debate
- MD03 - Resolución de problemas y estudio de casos prácticos
- MD04 - Prácticas en sala de informática
- MD05 - Seminarios
- MD07 - Realización de trabajos en grupo
- MD08 - Realización de trabajos individuales

ASSESSMENT METHODS (Instruments, criteria and percentages)

ORDINARY EXAMINATION DIET

The systems for skills audit and competences evaluation will preferably be based on the continuous assessment of students. This will be diversified and it will employ assessment techniques adapted during the course in order to verify the students skills audit and acquisition of competences. It will employ written exercises, problem solving, reports and oral expositions about the course contents, either individualised or in small groups.

The written exercises will focus on the theoretical contents of the course. The three proposed

written exercises will weight the 45% of the student final mark, 15% for each exercise.

Codding tasks, reports and oral expositions will represent 45% of the student final mark.

Finally, attendance and active involvement in the clases will represent 10% of the student final mark.

Assessment regulation at UGR are available in the "UGR Assessment Policy and Regulations" (BOUGr 112, 9 de Noviembre de 2016).

EXTRAORDINARY EXAMINATION DIET

Students who have not passed a course in the ordinary assessment session (convocatoria ordinaria) will have access to an extraordinary assessment session (convocatoria extraordinaria). This will follow the same guidelines that the single final assessment (see next).

SINGLE FINAL ASSESSMENT (evaluación única final)

Students who are unable to follow continuous assessment methods due to justifiable reasons shall have recourse to a single final assessment (evaluación única final). In order to opt for a single final assessment (evaluación única final), students must send a request to the Applied Mathematics Department. This has to be done in the first two weeks of the course, citing and justifying the reasons why they are unable to follow the continuous assessment system.

Single final assessment (evaluación única final), which is an assessment method that only takes a final exam into account, will be divided in a written exercise (60% of their mark) and a computing exercise (40% of their mark).

ADDITIONAL INFORMATION

In accordance with the Assessment Policy and Regulations (BOUGr 112, 9 de Noviembre de 2016) the students can have access either to a continuous assessment (evaluación continua) or to a single final assessment (evaluación única final) using the corresponding procedure described in this regulation.

In line with the CRUE and Secretariat for Inclusion recommendations the systems for skills audit and acquisition of competences considered in this course guide will be applied following to the principle of design for any student, making easier the learning and verification of knowledge in accordance with the students requirements and functional diversity.