

MODULE	SUBJECT MATTER	YEAR	SEMESTER	CREDITS	TYPE
Quantitative methods	Mathematics for Economics 1	1º	2º	6	Compulsory
TEACHING STAFF <sup>(1)</sup>			FULL CONTACT INFORMATION FOR OFFICE HOURS (Address, telephone number, email, etc.)		
<ul style="list-style-type: none"><li>• Antonia Delgado Amaro: group A (English)</li><li>• Joaquín F. Sánchez Lara: group B</li><li>• José Miguel Alonso Alonso: group C</li><li>• M<sup>a</sup> Victoria Fernández Muñoz: group D</li></ul> <p>COORDINATOR OF THE COURSE: Joaquín F. Sánchez Lara</p>			Department of Applied Mathematics, Faculty of Economics and Business. <ul style="list-style-type: none"><li>• Antonia Delgado Amaro: amdelgado[at]ugr.es, 958242947, offices B04 and 2.57 at Faculty of Sciences, Mathematics building, 2nd floor.</li><li>• Joaquín F. Sánchez Lara: jslara[at]ugr.es, 958241934, office B02.</li><li>• José Miguel Alonso Alonso: jmaa[at]ugr.es, 958248293, office B06.</li><li>• M<sup>a</sup> Victoria Fernández Muñoz: mvfm[at]ugr.es, 958241596, offices B04 and 26 at ETS Ing. Edificación, 5th floor.</li></ul>		
			TIMETABLE FOR TUTORIALS OR LINK TO WEBSITE		
			<a href="http://vvv.ugr.es">http://vvv.ugr.es</a> and <a href="https://mateapli.ugr.es/">https://mateapli.ugr.es/</a>		
BELONGS TO UNDERGRADUATE DEGREE PROGRAMME			AND ALSO TO OTHER UNDERGRADUATE DEGREE PROGRAMMES		
Grado in ECONOMICS			Finance and Accounting; Marketing and Market Research		
PREREQUISITES OR RECOMMENDATIONS (where applicable)					
Completion of the Mathematics course.					
BRIEF DESCRIPTION OF CONTENT (ACCORDING TO OFFICIAL VALIDATION REPORT)					

<sup>1</sup> Consult any updates in Acceso Identificado > Aplicaciones > Ordenación Docente

<sup>(∞)</sup> This course guide should be filled in according to UGR regulations on assessment of student learning: (<http://secretariageneral.ugr.es/pages/normativa/fichasugr/ncg7121/>)



- Real quadratic forms.
- Introduction to mathematical programming. The graphical method.
- Differential calculus for functions of several variables. Economic applications.
- Classic optimization without restrictions. Convex optimization.
- Integral calculus of functions of several variables.
- Introduction to differential equations.

#### GENERAL AND SPECIFIC COMPETENCES

- CB1: That students have demonstrated to possess and understand knowledge in an area of study that starts from secondary education, and is usually found at a level that, although supported by advanced textbooks, also includes some aspects that imply knowledge coming from the vanguard of its field of study.
- CB2: Make the students able to apply their knowledge to their work or vocation in a professional manner, and provide them with the skills to elaborate and defend their position and with the problem-resolution abilities in their area of application.
- CB3: That students have the ability to gather and interpret relevant data (usually within their area of study) to make judgments that include a reflection on relevant issues of social, scientific or ethical nature.
- CB5: That students have developed those learning skills necessary to undertake further studies with a high degree of autonomy.
- CG2: Cognitive comprehension skills.
- CG3: Capacity for analysis and synthesis.
- CG4: Ability of organization and planning.
- CG8: Ability in solving problems.
- CG9 - Ability in decision making.
- CG16: Ability in critical and self-critical reasoning.
- CG17: Learning capacity and autonomous work.
- CT1: Make the student able to identify and anticipate relevant economic problems related to resource allocation in general -both in the private and public arena- using the knowledge and concepts provided in the degree.
- CE21: Use basic tools of a quantitative nature for the diagnosis and economic analysis.
- CE22: Provide rationality to the analysis and description of any aspect of economic reality.
- CE23: Evaluate the consequences of alternative courses of action and select the best options given the objectives previously set.
- CE32: To be able to fluently communicate in an environment where team work applies.
- CE37: Mathematics optimization.
- CE50: Get skill in solving optimization problems in the economic field.
- CE51: Know techniques of differential and integral calculus in several variables and their applications in economics analysis.
- CE52: Know, understand and apply different methods of mathematical optimization as well as some of the main dynamical models in economics.
- CE53: Learn how to classify quadratic forms and to use graphical techniques to solve math programs and their applications in economics.
- CE54: Know the methods of solving first order differential equations as well as the techniques of differential and integral calculus in several variables and their application to economic analysis.

#### OBJECTIVES (EXPRESSED AS EXPECTED LEARNING OUTCOMES)

- Understand the concepts of partial derivatives, gradient vector and Hessian matrix of real functions of several variables.



- Learn the use of Taylor's theorem for approximating functions.
- Calculate the partial derivatives of a function implicitly defined.
- Understand the importance of the homogeneity of a function for economic applications.
- Calculate the local extreme of real functions of several variables.
- Mathematically formulate economic optimization problems.
- Graphically solve math programs in two variables.
- Study the convexity of a program and apply it to the calculation of global extremes.
- Calculate double integrals on simple regions.
- Calculate solutions of simple differential equations by the method of separation of variables.

## DETAILED SYLLABUS

### THEORY:

- **Topic 1. BASIC NOTIONS ON FUNCTIONS OF SEVERAL VARIABLES.**  
Notation for subsets of  $\mathbb{R}^n$ . Graphic representation of subsets of  $\mathbb{R}^2$ . Euclidean distance. Basic topology in  $\mathbb{R}^n$ : balls, relative position between points and sets (interior, exterior and boundary points), bounded, open, closed and compact sets. Basic notions on functions of several variables: domain, maximal domain and range. Operations with functions. Types of functions: separate variables, polynomial and rational functions. Quadratic forms: definition and classification. Level and sub-level sets. Coercive and anti-coercive functions. Some functions of several variables outstanding in economics: utility function, cost quadratic function, production function.
- **Topic 2. OPTIMIZATION WITH INEQUALITY RESTRICTIONS: THE GRAPHIC METHOD.**  
Definition of local and global extrema. The Weierstrass Theorem. Optimization with inequality restrictions: the graphical method in two variables. Problems of linear programming in two variables applied to the economic field.
- **Topic 3. DIFFERENTIAL CALCULUS FOR FUNCTIONS OF SEVERAL VARIABLES. OPTIMIZATION WITHOUT RESTRICTIONS.**  
First order partial derivatives. Gradient vector. Chain rule. Implicit derivation. Second order partial derivatives. Schwartz property. Hessian matrix. Taylor formula: lineal and quadratic approximation of functions. Critical points. Necessary and sufficient condition for local extrema. Saddle points. Convex and concave functions: properties. Sufficient conditions for existence of global extrema. Applications to maximization of benefit functions and minimization of cost functions.
- **Topic 4. INTEGRAL CALCULUS FOR FUNCTIONS OF SEVERAL VARIABLES.**  
Different types of definite integrals. Double integrals over rectangular regions. The Fubini's Theorem.
- **Topic 5. ORDINARY DIFFERENTIAL EQUATIONS.**  
Basic methods for resolution of first order differential equations. Separation of variables method. Economics models: classic models, investment and public spending models.

### PRACTICE:

#### Seminars/Workshops

- Workshop for contents reinforcement.

#### Laboratory work

- Session 1. Functions of several variables: definition and graphical representation.
- Session 2. Differential calculus for functions of several variables. Optimization without restrictions.



- Session 3. Resolution of double integrals over rectangular regions. Introduction to Ordinary Differential Equations.

## BIBLIOGRAPHY

### BASIC READING LIST

- M. Álvarez de Morales Mercado y M.A. Fortes Escalona. *Matemáticas Empresariales*. Ed. Copicentro.
- J. García Cabello. *El Cálculo Diferencial de las Ciencias Económicas*. Ed. Delta Publicaciones.
- E.F. Haeussler y R. Paul. *Matemáticas para Administración, Economía, Ciencias Sociales y de la Vida*. Ed. Prentice Hall.
- E.F. Haeussler and R. Paul. *Introductory mathematical analysis : for business, economics and the life and social sciences* (9th ed.). Upper Saddle River, N.J.: Prentice Hall.
- K. Sydsaeter, P. Hammond y A. Stom, *Matemáticas para el Análisis Económico*. Ed. Prentice Hall.
- K. Sydsaeter, P. Hammond, and A. Stom. *Essential mathematics for economic analysis*. Harlow: Pearson Education Limited.

### COMPLEMENTARY READING

- P. Alegre. *Matemáticas Empresariales*. Ed. AC.
- A. Balbás. *Análisis Matemático para la Economía (I y II)*. Ed. AC.
- A. Balbás. *Programación Matemática*. Ed. AC.
- R. Caballero. *Matemáticas Aplicadas a la Economía y la Empresa*. Ed. Pirámide.
- E. Costa. *Matemáticas para Economistas*. Ed. AC.
- G. Gandolfo. *Economic Dynamics*. Ed. Springer.
- J. García Hernández, C. Martínez Álvarez, M. L. Rodríguez González, *Optimización Matemática aplicada a la Economía*, Ed. Godel Impresiones Digitales S.L.
- H. Lomelí. *Métodos Dinámicos en Economía*. Ed. Thomsom.
- V. Ramírez González. *Matemáticas con Mathematica para Empresariales y Económicas*. Ed. Proyecto Sur.
- O. Samamed. *Matemáticas I. Economía y empresa*. Ed. Centro de Estudios Ramón Areces.
- O. Samamed. *Problemas Resueltos de Matemáticas I. Economía y Empresa*. Ed. Centro de Estudios Ramón Areces.
- D.G. Zill. *Ecuaciones Diferenciales con Aplicaciones*. Grupo Editorial Iberoamericano.

## RECOMMENDED LINKS

- Teaching platform MATEMAPLI: <http://vvv.ugr.es>
- Teaching platform PRADO: <http://www.ugr.es/estudiantes/prado>
- Web site of the Department of Applied Mathematics: <https://mateapli.ugr.es/>

## TEACHING METHODOLOGY

Teaching methodology includes:

- Master classes.
- Study and individual work by the student, including the search, checking and treatment of the necessary information sources, solving the problems and case studies and preparing class presentations.
- Individual or in group academic tutorials and evaluation test.

Face-to-face teaching consists of explaining the theoretical topics together with an interactive dialogue and the clarification of doubts (tutoring activities). This will provide the elements of analysis and reflection necessary for the



good understanding of the subject.

Practical activities will take place in classrooms and computer rooms. These are compulsory for the good understanding of the fundamentals. They consist of individually (and eventually in group) resolving practical problems as well as selected readings in relation to each topic raised that will serve to illustrate the contents exposed in class by the professor, allowing thus the analysis and the debate.

In case of monographic works, through individual (and eventually in group) follow-up tutorials, the teacher will supervise the learning process in order to ensure the assimilation of contents and the acquisition of the corresponding skills.

#### ASSESSMENT (ASSESSMENT INSTRUMENTS, CRITERIA AND PERCENTAGE VALUE OF FINAL OVERALL MARK, ETC.)

##### ORDINARY ASSESMENT

According to the Rules for Assessment and grading of the students of the University of the assessment of students' academic performance will reflect public, objective and impartial criteria, and will preferably be continuous. Nevertheless, the students may apply for a **single final assessment** (article 8 of the current Rules for Assessment, which provides for the taking of a single final assessment). On one hand, lack of application for single final assessment option will be understood as a waiver of the right of such assessment. On the other hand, those students who are granted with single final assessment are not eligible for continuous assessment.

In the continuous assessment option, attendance to the corresponding assessment activities is obligatory. Lack of attendance to the assessment activities on the dates and the places specified for it will be understood as a waiver of the right of performance of these activities. Dates and places for assessment activities will be made public sufficiently in advance.

In the **continuous assessment option**, the total score will be the sum of all scores corresponding to assessment activities. These are the following:

- *Diverse activities*: the continuous work of the student will be assessed through various activities which could consist of online tests, level test, partial exams, individual or in group works, workshops, face-to-face problem solving, computer practices with mathematical software,... and all those activities considered appropriate by the professor. The total of these activities scores a maximum of 50% of the total score.
- *Final exam*: A final written exam which scores a maximum of 50% of the total score. Date and place for the final written exam will be made public by the Faculty of Economic and Business Sciences.

Students with no attendance to the final exam will have the final mark "Not Having Been Submitted" ("No Presentado").

##### EXTRAORDINARY ASSESSMENT:

It will consist of a single written exam which will be graded on a 0-10 scale (scoring a maximum of 10 points). In order to pass the course under this option, a final mark equal or bigger than 5 is required. Otherwise, the course is considered to be failed. Date and place for the final written exam will be made public by the Faculty of Economic and Business Sciences.

Students with no attendance to such final written exam (that scores a maximum of 10 points) will have the final mark "Not Having Been Submitted" ("No Presentado").

#### DESCRIPTION OF THE EXERCISES WHICH WILL CONSTITUTE SINGLE FINAL ASSESSMENT AS ESTABLISHED IN UGR REGULATIONS



Following the regulations, a final evaluation is established for those students who have completed the required requirements and have applied for the single final assessment (<https://sede.ugr.es/sede/catalogo-de-procedimientos/solicitud-evaluacion-unica-final.html>).

It will consist of a single written exam which will be graded on a 0-10 scale (scoring a maximum of 10 points). In order to pass the course under this option, a final mark equal or bigger than 5 is required. Otherwise, the course is considered to be failed. Date and place for the final written exam will be made public by the Faculty of Economic and Business Sciences.

Students with no attendance to such final written exam (that scores a maximum of 10 points) will have the final mark "Not Having Been Submitted" ("No Presentado").

Students may apply for single final assessment option, either within the first two weeks of teaching of the subject or within two weeks following change of matriculation. Application is to be made through the electronic system (<https://sede.ugr.es/sede/catalogo-de-procedimientos/solicitud-evaluacion-unica-final.html>), citing and accrediting the reasons for not being able to undergo the system of continuous assessment (reasons of employment, health, disability or any other correctly justified cause).

### SCENARIO A (ON-CAMPUS AND REMOTE TEACHING AND LEARNING COMBINED)

#### TUTORIALS

**TIMETABLE** (According to Official Academic Organization Plan)

#### TOOLS FOR TUTORIALS

(Indicate which digital tools will be used for tutorials)

Available at

<https://mateapli.ugr.es>  
<http://vvv.ugr.es>

With few exceptions, the tutorial sessions will be attended by videoconference (through any telematic tool authorized by UGR) or official e-mail.

Individual tutorial sessions will take place upon request of the student. Some group tutorial sessions, compulsory or optional, will take place if they are considered appropriate as a training return tool in case virtual classes have to be given in an asynchronous way.

#### MEASURES TAKEN TO ADAPT TEACHING METHODOLOGY

- Following the guidelines included in the Plan for the adaptation of education in the 2020-2021 academic year to the health measures derived from the Covid-19 pandemic (approved in the Governing Council UGR 25-06-20), education in Scenario A is defined as a multimodal or hybrid education system that combines the greatest possible attendance with online classes (synchronous sessions) and non-attendance training activities for the autonomous learning of students.
- The distribution between face-to-face and non face-to-face activities, as well as the way they are carried out, will follow the guidelines set by the Faculty of Economic and Business Sciences, the UGR, and the health authorities.
- Tasks and exercises deliveries will be done through teaching platforms authorized by the UGR.
- As an additional measure, special attention would be given to providing teaching materials to students through a teaching platform authorized by the UGR.

#### MEASURES TAKEN TO ADAPT ASSESSMENT (Instruments, criteria and percentage of final overall mark)

##### Ordinary assessment session





The assessment system will preferably be continuous. The activities will be the same and with the same scores as in the normal attendance scenario.

If the health situation allows it, the tests and exams will be carried out in face-to-face mode. If this is not possible, they will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

#### Extraordinary assessment session

The extraordinary assessment session will consist of a single written exam whose score will be 100% of the final grade (10 points)

If the health situation allows it, the exam will be carried out in face-to-face mode. If this is not possible, it will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

#### Single final assessment

The single final assessment session will consist of a single written exam whose score will be 100% of the final grade (10 points)

If the health situation allows it, the exam will be carried out in face-to-face mode. If this is not possible, it will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

### SCENARIO B (ONCAMPUS ACTIVITY SUSPENDED)

#### TUTORIALS

**TIMETABLE** (According to Official Academic Organization Plan)

**TOOLS FOR TUTORIALS**

(Indicate which digital tools will be used for tutorials)

Available at

<https://mateapli.ugr.es>

<http://vvv.ugr.es>

Tutorial sessions will be attended by videoconference (through any telematic tool authorized by UGR) or official e-mail.

Individual tutorial sessions will take place upon request of the student.

Some group tutorial sessions, compulsory or optional, will take place if they are considered appropriate as a training return tool in case virtual classes have to be given in an asynchronous way.

#### MEASURES TAKEN TO ADAPT TEACHING METHODOLOGY

- All classes would be performed in virtual mode. Virtual classes will be given using Google Meet or any other authorized platform. Synchronous teaching will be preferable, although health circumstances may impose an asynchronous scenario. In this case classes will be recorded and shared by Google Drive, and will be complemented by specific follow-up and feed-back training actions proposed for that purpose (tutorials, tasks, exercise deliveries,...).
- Tasks and exercises deliveries will be done through teaching platforms authorized by the UGR.
- As an additional measure, special attention would be given to providing teaching materials to students through a teaching platform authorized by the UGR.



## MEASURES TAKEN TO ADAPT ASSESSMENT (Instruments, criteria and percentage of final overall mark)

### Ordinary assessment session

The assessment system will preferably be continuous. The activities will be the same and with the same scores as in the normal attendance scenario.

The activities will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

### Extraordinary assessment session

The extraordinary assessment session will consist of a single written exam whose score will be 100% of the final grade (10 points)

It will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

### Single final assessment

The single final assessment session will consist of a single written exam whose score will be 100% of the final grade (10 points)

It will be performed as deliveries of answers and solutions to problems. These deliveries will be carried out through educational platforms and telematic tools authorized by the UGR.

